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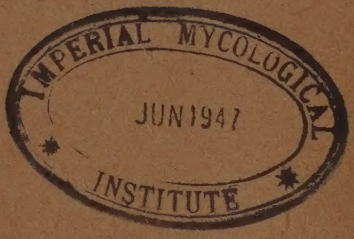

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MATHÉMATIQUES

SUR LES ÉQUATIONS DU TYPE DE STURM-LIOUVILLE ET LES FONCTIONS «POSITIVES»

Par A. POWSNER

(Présenté par S. N. Bernstein, de l'Académie, le 20. XI. 1943)

Considérons l'équation

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - [\rho(x) - \rho(y)]\, u = 0 \quad (-\infty < x,\ y < \infty) \tag{1}$$

Nous supposerons dans la suite que $\rho(x)$ est une fonction paire continue sur tout l'axe ox et que

$$\rho(x) = O(1/x^{3+\varepsilon}) \text{ pour } x \to \infty \quad (\varepsilon > 0)$$

Cherchons une fonction $u(x, y)$ qui présente dans tout le plan une solution de l'équation (1) et qui vérifie les conditions limites

$$u(x, 0) = f(x), \quad \frac{\partial u}{\partial y}(x, y)_{y=0} = 0 \quad (-\infty < x < \infty)$$

où $f(x)$ est une fonction paire donnée.

En vertu du théorème d'unicité il correspond à chaque fonction f une solution bien déterminée de l'équation (1), soit $u(x, y)f$. La fonction $u(x, y)f$ est un certain opérateur linéaire par rapport à f; nous le désignerons par $T_x^y(f)$

$$u(x, y)f = T_x^y f(x).$$

Choisissons pour axes de coordonnées les caractéristiques de l'équation (1). Quand on fait le changement des variables défini par les formules $x = \frac{\xi + \eta}{\sqrt{2}}$, $y = \frac{\eta - \xi}{\sqrt{2}}$ l'équation (1) prend la forme

$$\frac{\partial^2 u}{\partial \xi\, \partial \eta} - \frac{1}{2}\left[\rho\left(\frac{\xi + \eta}{\sqrt{2}}\right) - \rho\left(\frac{\eta - \xi}{\sqrt{2}}\right)\right] u = 0. \tag{1'}$$

Les conditions limites du problème deviendront

$$(u\,\xi, \xi) = f(\sqrt{2}\xi); \quad \frac{\partial u}{\partial \eta} = \frac{\partial u}{\partial \xi} \quad (\xi = \eta).$$

Maintenant pout résoudre l'équation (1'), nous utiliserons la fonction de Riemann; nous aurons alors

$$(u\,\xi_0, \eta_0) = \frac{1}{2}[f(\sqrt{2}\,\xi_0) + f(\sqrt{2}\,\eta_0)] - \int_{A_1}^{A_2} (P\,d\eta - Q\,d\xi) \tag{2}$$

où A_1 et A_2 sont les points d'intersection des droites $\eta=\eta_0$, $\xi=\xi_0$ avec la droite $\xi=\eta$, et l'intégrale est prise le long de la droite $\xi=\eta$

$$P=\frac{1}{2}\left[v(\xi,\eta;\xi_0,\eta_0)\frac{\partial u}{\partial\eta}-u\frac{\partial v}{\partial\xi}(\xi,\eta;\xi_0,\eta_0)\right],$$

$$Q=\frac{1}{2}\left[v\frac{\partial u}{\partial\xi}-u\frac{\partial v}{\partial\eta}\right]$$

et $v(\xi,\eta;\xi_0,\eta_0)$ est la solution de l'équation (1'), si l'on exige que

$$v(\xi_0,\eta;\xi_0,\eta_0)=1,\quad v(\xi,\eta_0;\xi_0,\eta_0)=1.$$

En calculant l'expression (2) on trouve

$$u(\xi_0,\eta_0)=\frac{1}{2}[f(\sqrt{2}\,\xi_0)+f(\sqrt{2}\,\eta_0)]-\frac{1}{2\sqrt{2}}\int_{\eta_0\sqrt{2}}^{\xi_0\sqrt{2}} f(t)\left(\frac{\partial v}{\partial\eta}-\frac{\partial v}{\partial\xi}\right)_{\xi=\eta=t/\sqrt{2}}dt \qquad (3)$$

En revenant aux coordonnées x, y et en introduisant naturellement une nouvelle fonction $\tilde{w}(t;x,y)$, nous aurons pour $u(x,y)$ une expression de la forme*

$$T_x^y f(x)=u(x,y)=\frac{1}{2}[f(x+y)+f(x-y)]-\int_0^\infty f(t)\,\tilde{w}(t;x,y)\,dt \qquad (4)$$

Théorème 1. *On a les identités*

$$\left.\begin{array}{c} u(-x,y)f=u(x,-y)f=u(x,y);\ u(x,y)=u(y,x) \\ T_t^\eta T_t^\xi f(t)=T_t^\xi T_t^\eta f(t)\end{array}\right\} \qquad (5)$$

La démonstration du théorème est basée sur ce fait que chacune des formules (5) est équivalente à une certaine identité pour la fonction $\tilde{w}$. Après avoir vérifié chacune des formules (5) pour un ensemble spécial de fonctions f, qui donne une approximation au sens de la convergence uniforme dans chaque intervalle fini pour chaque fonction continue (un tel ensemble sera indiqué dans la suite), nous démontrerons ainsi que l'identité considérée est vraie.

Les opérateurs $T_x^y f$ de cette nature ont été considérés par Delsarte(1) et ensuite par Lewitan (2) dans le cas où $\rho(x)$ est deux fois, et $f(x)$ quatre fois dérivable.

Dans la suite nous allons imposer la condition suivante à la famille des opérateurs $T_x^y f(x)$: Il existe une constante K qui ne dépend pas de x et y et telle que

$$\sup_{\substack{-\infty<x<\infty\\-\infty<y<\infty}}|T_x^y f(x)|\leqslant K\sup_{-\infty<x<\infty}|f(x)| \qquad (A)$$

Théorème 2. *Pour que la relation* (A) *soit vraie, il faut et il suffit qu'il existe une constante M telle que*

$$\int_0^\infty |w(t;x,y)|\,dt\leqslant M$$

pour x et y quelconques.

La démonstration est évidente.

Dans la suite nous allons considérer l'équation (1) dans une forme un peu modifiée

$$\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}-c[\rho(x)-\rho(y)]u=0 \qquad (1'')$$

---

* Au moyen de l'expression (4) l'opérateur $T_x^y f(x)$, qui a été précédemment défini seulement pour les fonctions f deux fois dérivables, s'étend naturellement à l'ensemble de toutes les fonctions sommables dans chaque intervalle fini. C'est dans cet espace que nous allons le considérer dans la suite.

Si c est fixe, nous obtenons une famille d'opérateurs $T_x^y f(x)$. Quelles sont les conditions nécessaires et suffisantes pour que cette famille vérifie la condition (A)? Considérons l'équation

$$L(\varphi) = \frac{d^2\varphi(x)}{dx^2} - c\rho(x)\varphi(x) = 0 \quad (0 \leqslant x < \infty)$$

Elle possède deux solutions linéairement indépendantes $\varphi_1(x)$ et $\varphi_2(x)$ et d'ailleurs

$$\varphi_1(x) - 1 = O(1/x^{1+\varepsilon}),\ \varphi_1'(x) = O(1/x^{2+\varepsilon})$$
$$\varphi_2(x) - x = O(1/x^{1+\varepsilon}),\ \varphi_2'(x) - 1 = O(1/x^{2+\varepsilon})$$

pour $x \to \infty$.

Théorème 3. *Pour que la relation* (A) *ait lieu, il faut et il suffit que* $\varphi_1'(0) = 0$.

Comme $\varphi_1(x)$ est une fonction entière de c, la condition (A) aura lieu dans le cas et dans ce cas seulement où c est un zéro d'une certaine fonction entière. En supposant que $\rho(x)$ est non négative et qu'elle n'est pas identiquement nulle, et en modifiant légèrement les méthodes connues de la théorie de Sturm, on démontre facilement que les zéros de la fonction entière correspondante sont tous réels et qu'il y en a une infinité.

La démonstration du théorème 3 est basée sur une étude détaillée des propriétés de la fonction de Riemann pour l'équation (1″); nous ne l'exposerons pas ici.

Considérons maintenant l'espace K de toutes les fonctions paires sommables sur tout l'axe. Introduisons la multiplication dans K de la manière suivante: si $f \in K$, $g \in K$, nous poserons

$$f * g = \int_{-\infty}^{\infty} f(x-t)\, g(t)\, dt - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w(t; x, y)\, f(x)\, g(y)\, dx\, dy \qquad (6)$$

Ayant introduit dans K une norme, en posant $\|f\| = \int_{-\infty}^{\infty} \|f(t)\|\, dt$, nous verrons que les définitions de norme et de multiplication dans K sont telles que K est un anneau normé sans unité au sens de Gelfand (3) quand la condition (A) est vérifiée, et dans ce cas seulement. En effet, la condition (A) et le théorème 1 permettent de vérifier tous les axiomes. En introduisant formellement d'une certaine manière l'unité, nous obtiendrons, quand (A) est vérifiée, un anneau normé $\tilde{K}$ avec une unité. Dans la suite nous supposerons que $\tilde{K}$ est un anneau normé.

Soit $\varphi(s, \lambda)$ une fonction caractéristique de l'opérateur $L(\varphi) = \frac{d^2\varphi}{dx^2} - c\rho(x)\varphi(x)$, qui vérifie la condition $\varphi(0, \lambda) = 1$, $\varphi'(0, \lambda) = 0$, $L\varphi + \lambda^2\varphi = 0$ $(\lambda \geqslant 0)$.

Théorème 4. *A chaque idéal maximal* M *de l'anneau* $\tilde{K}$ *(sauf pour* $M = K$*) il correspond une et une seule fonction* $\varphi(x, \lambda)$ *telle que*

$$f(M) = \int_{-\infty}^{\infty} f(t)\varphi(t, \lambda)\, dt \quad (f \in K)$$

Inversement, chaque fonction $\varphi(t, \lambda)$ *définit un idéal maximal: c'est l'ensemble des éléments* $\alpha e + f \in \tilde{K}$ $(f \in K)$ *qui vérifient la condition*

$$\alpha + \int_{-\infty}^{\infty} f(t)\varphi(t, \lambda)\, dt = 0$$

Remarque. L'ensemble de toutes les combinaisons linéaires des fonctions $\varphi(x, \lambda)$ peut être considéré comme système des fonctions indiqué dans le théorème 1, si l'on tient compte de l'identité

$$T_x^y \varphi(x, \lambda) = \varphi(x, \lambda)\varphi(y, \lambda) \tag{7}$$

Il suit immédiatement du théorème 4 que $\sup_{-\infty<x<\infty} |\varphi(x, \lambda)| < h$, où h est une certaine constante qui ne dépend pas de λ. On en déduit un corollaire intéressant.

Corollaire 1. *Pour que toutes les fonctions $\varphi(x, \lambda)$ soient uniformément bornées sur tout l'axe et possèdent une borne commune, il faut et il suffit que la fonction $\varphi_1(x)$ vérifie la même condition initiale $\varphi_1'(0) = 0$.*

Ce résultat peut être obtenu sans utiliser la théorie des anneaux normés en partant de formule (7).

Les résultats obtenus nous permettent de généraliser le théorème de Bochner sur les fonctions «positives».

Nous dirons qu'une fonction $f(x)$ est L-positive si l'on peut la représenter dans la forme

$$f(x) = \int_0^\infty \varphi(x, \lambda)\, d\sigma(\lambda)$$

où $\sigma(\lambda)$ est une fonction non décroissante à variation bornée.

Théorème 5. *La fonction $f(x)$ est L-positive dans le cas et dans ce cas seulement où*

1) $f(x) = f(-x)$

2) $|f(x)| < M < \infty \quad (-\infty < x < \infty)$

3) $$\sum_{i=1}^{n}\sum_{k=1}^{n} \frac{1}{2}[f(x_i - x_k) + f(x_i + x_k)] z_i z_k + \sum_{i=1}^{n}\sum_{k=1}^{n} \int_0^\infty f(t) w(t; x_i, x_k) z_i z_k\, dt \geqslant 0$$

quels que soient $x_1, x_2, \ldots, x_n$.

Si $L(\varphi) = d^2\varphi/dx^2$, nous aurons une condition pour que $f(x)$ soit représentable dans la forme $f(x) = \int_0^\infty \cos \lambda x\, d\sigma(\lambda)$, c'est-à-dire le théorème de Bochner.

La démonstration du théorème peut être obtenue si l'on modifie légèrement la méthode de F. Riesz dans la forme qui lui est donnée dans la Note de D. Raikoff (4) et si l'on tient compte de ce que $\tilde{K}$ est un anneau normé.

Remarque. Il est impossible d'obtenir ici la condition 2) comme conséquence de la condition 3), ainsi que l'on fait en démontrant le théorème classique de Bochner, car même dans le cas où $p(x) = 0$ la fonction cos hyp x vérifie la condition 2). Indiquons encore quelques résultats qui peuvent être obtenus dans cet ordre d'idées. Considérons les équations

$$\text{a)}\ \frac{\partial^2 u}{\partial x^2} - c\,p(x)\,u = \frac{\partial^2 u}{\partial y^2}, \qquad \text{b)}\ \frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 \psi}{\partial y^2} - c\,p(x)\,\psi$$

avec les conditions limites $u(x, 0) = f(x)$, $\frac{\partial u}{\partial y}_{y=0} = 0$, où $f(x)$ est une certaine fonction paire. Comme dans le cas de l'équation (1), nous obtiendrons les solutions des équations a) et b) dans la forme

$$u(x, y) = \frac{f(x+y) + f(x-y)}{2} - \int_0^\infty f(t) w_1(t; x, y)\, dt \tag{8}$$

$$\psi(x, y)=\frac{f(x+y)+f(x-y)}{2}-\int_0^\infty f(t)w_2(t; x, y)\,dt \qquad (9)$$

où w_1 et w_2 sont certaines fonctions que l'on construit à partir des fonctions de Riemann pour les équations (8) et (9). Posons dans (8) et (9) $x=0$. Alors les formules

$$u(0, y)=Pf=f(y)+\int_0^\infty f(t)w_1(t; 0, y)\,dt \qquad (8')$$

$$\psi(0, y)=Qf=f(y)+\int_0^\infty f(t)w_2(t; 0, y)\,dt \qquad (9')$$

définissent dans l'espace des fonctions sommables dans chaque intervalle fini deux opérateurs Pf et Qf. En posant dans (8) $f(x)=\varphi(x, \lambda)$ et dans (9) $f(x)=\cos\lambda x$, nous aurons

$$u(x, y)=\varphi(x, \lambda)\cos\lambda y, \quad \psi(x, y)=\cos\lambda x\,\varphi(x, \lambda)$$

d'où

$$P\varphi(x, \lambda)=\cos\lambda x, \quad Q\cos\lambda x=\varphi(x, \lambda) \qquad (10)$$

En tenant compte de ces formules on peut démontrer que

$$PQf=QPf=f$$

Pour ces opérateurs on a le théorème 3′ analogue au théorème 3.

Théorème 3′. *Pour que les opérateurs P et Q soient bornés, il faut et il suffit que* $\varphi_1'(0)=0$.

Si les opérateurs P et Q sont bornés, il est évident que l'opérateur Q transforme chaque fonction paire presque-périodique au sens de Pohr en une fonction qui peut être approchée uniformément sur tout l'axe par des combinaisons linéaires des fonctions $\varphi(x, \lambda)$. Et inversement, l'opérateur P transforme chaque fonction qui peut être approchée uniformément sur tout l'axe par des combinaisons linéaires des fonctions $\varphi(x, \lambda)$ en une fonction paire presque-périodique. Cette remarque permet de construire une théorie analogue à celle des fonctions presque-périodiques, où le rôle des cosinus sera joué par les fonctions $\varphi(x, \lambda)$.

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LITTÉRATURE CITÉE

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MATHEMATICAL PHYSICS

SOLUTION OF HOMOGENEOUS LINEAR EQUATIONS BY ITERATION METHOD

By N. M. HOPSTEIN

(Communicated by V. A. Fock, Member of the Academy, 29. II. 1944)

We here propose a method of iterations for the solution of linear homogeneous equations and consider the conditions of convergence of the process.

In the problem of small oscillations of mechanical systems there occur systems of homogeneous linear equations of form

$$(\lambda - a_{ii})x_i - \sum_{k \neq i} a_{ik}x_k = 0 \qquad i, k = 1, 2, \ldots, n \tag{1}$$

As is known, this system has a non-zero solution if

$$D(\lambda) = \begin{vmatrix} \lambda - a_{11} & -a_{12} \ldots -a_{1n} \\ -a_{21} & \lambda - a_{22} \ldots -a_{2n} \\ \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots \end{vmatrix} = 0 \tag{2}$$

Equality (2) is an equation of the n-th degree with respect to λ determining n characteristic values of λ. The procedure of expanding the determinant and solving the equation obtained is rather laborious and therefore, when n is large, impracticable. The unique approximate method, that of successive diagonalizations due to Jacobi(1) and improved by Eliashevich and Stepanov(2), also involves tedious computations. For the solution of the problem the method of iterations can be applied in the following form. We write the system (1) in the form

$$\lambda = a_{11} + \sum_{2}^{n} a_{1k}x_k \tag{3}$$

$$-a_{i1} = a_{i2}x_2 + \ldots + (a_{ii} - \lambda)x_i + \ldots + a_{in}x_n \tag{4}$$

where we put $x_1 = l$, all x_i being determined up to a constant factor.

We take a value $\lambda = \lambda_0$ as the initial approximation. Substituting it in (4) and solving the system of equations obtained, we get the approximate values $x_{i(0)}$. Introducing them in (3), we come to an improved value $\lambda = \lambda_1$ with which we proceed as with λ_0, *etc.* We thus obtain the sequences λ_0, λ_1,

$\lambda_2, \ldots$ and $x_{i(0)}, x_{i(1)}, \ldots$ which, under certain conditions, tend to a solution of the system (1).

A condition for convergence can be obtained as follows. One can show that if λ_0 and λ_1 are successive approximations, then

$$\lambda_1 = \lambda_0 - \frac{D(\lambda_0)}{D_{11}(\lambda_0)} \tag{5}$$

where D_{11} is the minor of D corresponding to the element $\lambda - a_{11}$. The iteration process for such an equation is convergent if

$$0 < \frac{\partial}{\partial\lambda}\frac{D(\lambda)}{D_{11}(\lambda)} < 2 \tag{6}$$

It is difficult to use the criterion in this form, because it requires that the minors of D should be computed. In virtue of the formula (5) it can be replaced by an approximate one:

$$0 < \left|\frac{\lambda_2 - \lambda_0}{\lambda_1 - \lambda_0}\right| < 2 \tag{7}$$

where $\lambda_0, \lambda_1, \lambda_2$ are three successive approximations.

Our method in the form exposed is, however, rather difficult, because to obtain every next approximation it is necessary to solve a system of equations in $n-1$ variables. The convergence can be considerably improved by the following method. The roots of the equation $D=0$ coincide with those of the equation

$$D/D_{11} = 0 \tag{8}$$

provided D has no multiple root and, therefore, $D_{11} \neq 0$. We can apply the middle ordinates method, useful for the solution of algebraic equations, to the equation (8).

Then by (5)

$$\lambda_x = \frac{\lambda_1^2 - \lambda_0\lambda_2}{2\lambda_1 - \lambda_0 - \lambda_2} \tag{9}$$

which is a much better approximation than λ_2. The value λ_x can be used again as the initial approximation for (3), (4).

The calculations can be considerably simplified by using the process of approximation with respect to n variables $\lambda, x_2, \ldots, x_n$ instead of that with respect to λ only. The slowness of the convergence of the process is offset by the simplicity of the calculations involved. Put the system (1) in the form

$$\lambda = a_{11} + \sum_2^n a_{1k}x_k$$

$$x_i = \frac{1}{a_{ii} - \lambda}\sum_{k\neq i} a_{ik}x_k \quad x_1 = 1 \tag{10}$$

At first we choose some initial values λ_0, $x_{k(0)}$, and the formulae (10) give us the improved values.

The conditions for convergence in this case are

$$\sum_2^n |a_{1k}| < 1$$

$$\frac{1}{|\lambda - a_{ii}|}\left(|x_i| + \sum_{k\neq i}|a_{ik}|\right) < 1 \tag{11}$$

Consider now a numerical example. Les us take the system

$$\begin{aligned}(1-\lambda)\,x_1+0.1\;x_2+0.3\;x_3&=0\\ 0.1\;x_1+(2-\lambda)\,x_2+0.2\;x_3&=0\\ 0.3\;x_1+0.2\;x_2+(3-\lambda)\,x_3&=0\end{aligned}$$

and solve it by the first method, taking

$$\lambda_0=1,\quad x_1=1$$

as initial approximation. Then (4) gives $x_{2(0)}=-0.07143$, $x_{3(0)}=-0.14286$. We see that the condition for convergence (11) is fulfilled.

We thus obtain

the 0-th appr.	$\lambda=1.000000$	$x_{2(0)}=-0.07143$	$x_{3(0)}=-0.14286$
» 1-st »	0.950000	−0.068639	−0.139695
» 2-nd »	0.951230	−0.0687049	−0.139722
» 3-nd »	0.951214	−0.0687039	−0.139721

The following approximations give the same result.

To obtain six correct significant figures we had to perform three operations. The formula (9) is useless in this case, the convergence being too rapid. Let us solve our system by the second method.

We take

$$\lambda=1.000000,\quad x_1=1,\quad x_2=0.000000,\quad x_3=0.000000$$

as the initial approximation. The next approximation, according to (10), is $x_2=-0.1$, $x_3=-0.15$. Using these values, we get $\lambda=0.945$.

We thus obtain

the 0-th appr.	$x_2=0.000000$	$x_3=0.000000$	$\lambda=1.000000$
» 1-st »	−0.100000	−0.150000	0.945000
» 2-nd »	−0.0663580	−0.136253	0.952489
» 3-rd »	−0.0694498	−0.140374	0.950944
» 4-th »	−0.0685617	−0.139630	0.951255
» 5-th »	−0.0687246	−0.139738	0.951206
» 6-th »	−0.0687005	−0.139720	0.951214
» 7-th »	−0.0687039	−0.139724	0.951214
» 8-th »	−0.0687039	−0.139722	0.951214

To attain six significant figures we had to perform six operations for λ and eight operations for x_2, x_3. Taking $\lambda=2$, $x_2=1$ and $\lambda=3$, $x_3=1$ we can obtain the remaining systems of solutions. The number of operations required depends essentially on the choice of the initial approximation. When the conditions for convergence are fulfilled, as it takes place in the example above, we may adopt

$$\lambda_0=a_{11},\quad x_1=1,\quad x_{k(0)}=0$$

as initial approximation. Taking for x_1 all the x's one after another, we obtain all solutions of the system.

The problem becomes considerably more complicated in the case where two or several roots have a strong resonance. This case was studied closely by Eliashevich in (3).

As it was established, in this case the system can be split into several systems in less than n variables by a linear transformation of variables. In the

case where coefficients a_{ij} are close to one another, we may consider our system as possessing an approximate symmetry. Applying a convenient transformation of variables, we sometimes obtain a system satisfying the conditions for convergence. In particular, when the coefficient of x_1 is close to that of x_2, we use the transformation

$$y_1 = x_1 + x_2, \qquad y_2 = x_1 - x_2 \tag{12}$$

When the coefficients of x_1, x_2, x_3 are close to one another, we use the transformation

$$y_1 = x_1 + x_2 + x_3, \quad y_2 = 2x_1 - x_2 - x_3, \quad y_3 = x_2 - x_3 \tag{13}$$

For instance, the iteration process being applied directly to the system

$$\begin{aligned} (1-\lambda)x_1 + 0.2\ x_2 + 0.2\ x_3 &= 0 \\ 0.2\ x_1 + (0.95-\lambda)x_2 + 0.2\ x_3 &= 0 \\ 0.2\ x_1 + 0.2\ x_2 + (1.05-\lambda)x_3 &= 0 \end{aligned}$$

does not converge. The transformation (13) gives

$$\begin{aligned} (1.4-\lambda)y_1 - 0.05\ y_3 &= 0 \\ -0.2\ y_1 + (0.933\ldots-\lambda)y_2 + 0.0166\ldots y_3 &= 0 \\ -0.033\ldots y_1 + 0.0166\ldots y_2 + (0.8-\lambda)y_3 &= 0 \end{aligned}$$

This system satisfies the condition for convergence, and the usual iteration process can, therefore, be applied. The advantage of our method in comparison with that of Jacobi is that it gives not only λ but also x_i without any additional calculations.

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PHYSICS

SOFTENING TEMPERATURE OF POLYMERS

By A. P. ALEXANDROV, Corresponding Member of the Academy,
and J. S. LAZURKIN

The softening of polymers is a matter of great importance in technology and also determines the possible working conditions for articles made from polymeric substances. The frost stability of rubber and the thermostability of plastics are determined mainly by the softening temperature.

As has been shown earlier (1), the alteration of the properties of simple and polymeric amorphous bodies upon softening is not connected with the phase transition, but is rather due to an increase of the relaxation velocity. In simple amorphous bodies the molecular re-groupings are not limited, and the increase of the relaxation velocity reveals itself in a decrease of the viscosity. This accounts for the change of their mechanical properties in the softening range.

In the case of polymeric substances one must distinguish the re-grouping of whole macromolecules and the re-grouping of the links of a polymeric chain. The latter is limited by the fact that the links are bound into macromolecules. Caused by external forces, the re-grouping of the links in macromolecules can lead to a change of the entropy and potential energy of the system (because the macromolecules change in shape). The elastic forces which will then develop hinder the deformation and return the system to the equilibrium state after removal of the external forces. The initial shape of the body is re-established more completely, the more difficult is the relative re-grouping among the macromolecules. Now the difficulty of this re-grouping, *i. e.* of the actual flow of the polymer, increases sharply with increasing length of the macromolecules. In the case of high polymers in contradistinction from simple amorphous bodies the fluidity above the softening temperature is very small. This in spite of the marked mobility of the links of the polymeric chain, which leads to a pronouncedly high elasticity.

The softening of polymers is thus connected with the increase in mobility of the links of the polymeric chains.

The object of the present paper consists in investigating the possible dependence of the softening temperature of the polymers on the degree of polymerization. The mobility of a link of a macromolecule under given external conditions (temperature, pressure) depends on its interaction with the surrounding medium and, in particular, with its neighbours in the same chain.

The conditions of the fastening of all links, sufficiently removed (3 to 5 links off) from the ends of the macromolecule are completely similar; this is a consequence of the short radius of action of the molecular forces and is brought into evidence by the fact that the combustion heat of the polymers is independent of their molecular weight if the degree of polymerization is large enough. The mobility of the links of isolated macromolecules if the degree of polymerization exceeds 15—20 must therefore be independent of the degree of polymerization.

The interaction between a given link of a polymeric chain and the surrounding medium is determined in first approximation by the density (at a given temperature and pressure).

In the case of polymerization when part of van-der-Waals' forces in the system is replaced by chemical forces, the density grows in accordance with the increase of the «concentration» of chemical bonds in the system. The concentration of chemical bonds is understood to mean the ratio of the number of chemical bonds of a given type, formed in the process of polymerization, to the initial number of monomeric molecules.

For lynear chain polymers with a degree of polymerization z the concentration of chemical bonds is

$$C = \frac{(z-1)n}{N}$$

where n denotes the number of macromolecules, while N is the total number of initial molecules. It is clear that for linear polymers which are homogeneously completely polymerized we have $N = nz$. For small z the concentration of chemical bonds in the system increases with z substantially; but when z has reached a range of about 15—20, C is near to its limit which is equal to 1; accordingly, the density of the polymer also rapidly approaches saturation with a rise of z.

The relaxation velocity, which is determined by the mutual action of a link of the chain with the surrounding medium (under given external conditions and temperature) can fall off with the increase of z for small values of z only, as long as the density and the conditions of fastening of the links in the chain are changing. When, however, C has approached saturation, the mobility of the links will not undergo any further alteration and the relaxation velocity must become constant.

On the assumption that the softening of the polymers is determined by the mobility of the links or of small portions of the chains but not of the entire chains, one may well expect the softening temperature to be independent of the length of the chains when the degree of polymerization z exceeds 15—20.

Indeed, the correctness of this conclusion is verified by the experimental results of Jenckel and Überreiter (3) who have studied the softening of polymers by dilatometric methods.

However, these measurements characterize the appearance of elasticity in an indirect way.

We have measured the dependence of the deformation of the polymers of styrol and methylmetacrilate on the temperature for a given period and amplitude of a force varying according to the sinusoidal law*.

After the polymerization process there usually remains in the polymer a more or less considerable number of monomers or of low molecular polymerization products which strongly decrease the softening temperature of the samples.

This circumstance has been often neglected while it accounts for the apparent dependence of the softening temperature on the molecular weight (which is determined according to Staudinger for the gross polymer without determining the quantity of the high-molecular fraction). This error lies at the basis of the now current view on the softening temperature and thermostability of plastics.

For this reason the procedure in all our experiments after the mechanical measurements had been made was as follows. The samples of styrol and methylmetacrilate were diluted, the high-molecular fraction was precipitated and dried *in vacuo*, wherafter the molecular weight and the content of the polymer in the sample were determined. The character of the dependence of the deformation on temperature is shown in Fig. 1. The rising portion of the curve was extrapolated up to the abs-

* The device is described in the paper by one of the authors(2).

cissa and the intersection point was identified with the softening temperature.

The softening temperature is here defined as that temperature, for which the relaxation time has a certain chosen value (of the order of the deformation period—1 min.).

The results of the measurements are represented in Fig. 2 for polymers with a molecular weight between 26 000 and 420 000 for styrol and between 20 000 and 135 000 for methylmetacrilate, containing different amounts of the monomer.

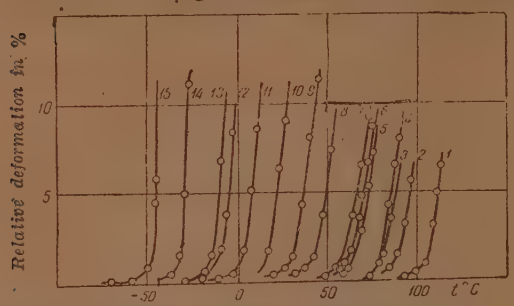

Fig. 1. Dependence of the deformation of the styrol sample on temperature. The numbers of curves correspond to the numbers of the points in Fig. 2

It can be seen from Fig. 2 that the softening temperature of the various samples is independent of the molecular weight of the polymer and the difference between the softening temperatures is uniquely determined by the content of the low-molecular fraction (monomer).

The softening temperature of the dimer of styrol, measured by the authors according to Tamman's method, lies at—80°, *i. e.* close to the softening of the polymer which contains 50 per cent of the monomer fraction. In general one may hardly expect these temperatures to coincide, since the dimer has a different structure and its softening, just as the softening of the monomer, is not connected with the inner mobility of the different parts of the molecule.

It seems to us very desirable to carry out detailed measurements of softening temperatures of low polymers (for z ranging from 3—5 to 50—100).

The measurement of the softening temperatures of two samples of sodium-

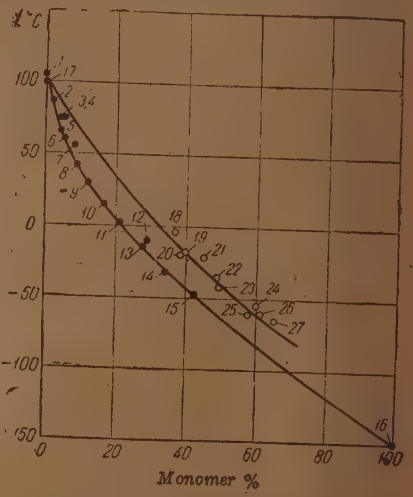

Fig. 2. Dependence of softening temperature of the samples of styrol and methylmetacrilate on the content of monomer.

Molecular weights:

Styrol:	1–420000	7–124000	12–420000
	2–420000	8– 79400	13– 94500
	3–118000	9– 69000	14–420000
	4– 26500	10–420000	15–420000
	5– 27000	11–420000	16–monomer
	6– 66400		
Methylmetacrilate:		17–123000	22–123000
		18–135000	23–123000
		19–123000	24– 24000
		20–123000	25–123000
		21– 20000	26–119000
			27–123000

divinyl caoutchouc of the same structure, with molecular weights 40 000 and 120 000, has also shown the same softening temperature, *viz.*—51°.

From the above-said it can be seen that the softening temperature of polymers with sufficiently high molecular weights (in the case of a similar structure) is independent of the molecular weight.

Thus, for a given structure of the linear polymer the frost stability of caoutchoucs and the thermostability of plastics cannot possibly be altered by changing the degree of polymerization, since in this case the concentration of chemical bonds in the system undergoes but a very slight alteration and the mobility of the links of the polymeric chain remains constant.

In the case of space polymerization the concentration of the chemical bonds can be considerably higher. The rise of the concentration of chemical bonds in the case of space polymerization leads accordingly to a decrease of the mobility of the links of the macromolecules.

It can, however, easily be seen that a small number of branchings (for instance, 2—5 per hundred macromolecule links, as in the case of vulcanized rubber) cannot essentially change the concentration of the bonds, being at the same time unable to change the softening temperature. If, however, the number of branchings approaches in the order of magnitude the number of fundamental molecules of the substance (as, for instance, in the case of the thermopolymer of divinyl—«escapone», and ebonite) the conditions of fastening of all the links of the chain are changed*, the density of the polymer increases markedly and so does also the softening temperature.

Conclusions

1. It has been shown that the softening temperature of sufficiently high polymers is independent of the degree of polymerization. This supports the conception according to which the softening of polymers is determined by the mobility of the different links or portions of the same polymeric chain but not by the mobility of the chains as a whole.

2. It can be assumed that the linear polymers with a given chemical composition possess a lower softening temperature than the branched and space polymers. A small number of branching cannot, however, alter the softening temperature markedly. An essential rise of the softening temperature can be expected in the case of a large number of branchings only.

A large part of the measurements, reported in the present paper, was carried out before the war by I.Firun and R. Panich, undergraduates of the Leningrad Polytechnical Institute.

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Academy of Sciences of the USSR.

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* If the space polymerization is carried out by introducing foreign substances, the picture becomes in general more complicated.

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PHYSIQUE APPLIQUÉE

SUR UN PHÉNOMÈNE D'INDUCTION DÛ À L'INTERACTION DE CHAMPS MAGNÉTIQUES PERPENDICULAIRES

Par I. BERSTEIN

(Présenté par L. I. Mandelstam, de l'Académie, le 8. II. 1944)

Cette note préliminaire contient la théorie schématique d'un phénomène observé par l'auteur en collaboration avec M-lles Goronina et Joukova, phénomène qui nous a permis de réaliser un nouveau type de magnétomètre («oerstedmètre»): un fil ferromagnétique rectiligne entouré d'une bobine dont les spires lui sont perpendiculaires étant placé dans un champ magnétique constant parallèle au fil, un courant alternatif parcourant celui-ci induit une f. e. m. alternative dans la bobine.

Cela ne serait pas le cas si la relation entre l'induction et le champ dans le fil était linéaire*, car alors le flux traversant la bobine ne varierait pas. Le phénomène résulte d'une interaction non-linéaire de deux champs magnétiques perpendiculaires: les variations de la composante transversale (par rapport à axe du fil) du champ, excitée par le courant, font varier la composante longitudinale de l'induction, excitée par le champ longitudinal constant**.

Une théorie schématique peut être facilement construite en faisant les idéalisations suivantes:

1. Les propriétés magnétiques du fil sont identiques dans toutes les directions.

2. On néglige l'hystéresis.

La première hypothèse suppose l'absence de tensions anisotropes ainsi qu'un rapport suffisant de la longueur du fil à son diamètre.

La seconde hypothèse peut être justifiée ainsi: notre fil ferromagnétique possède un cycle d'hystéresis de très petites dimensions, tant en largeur qu'en longueur; l'amplitude du champ dans la presque totalité du volume du fil dépasse largement la valeur correspondant aux extrémités du cycle d'hystéresis.

Les hypothèses ci-dessus admises, la relation entre l'induction $\mathbf{B}$ et le champ $\mathbf{H}$ dans le fil est de la forme

$$\mathbf{B} = f(H)\frac{\mathbf{H}}{H} \qquad (1)$$

* Le fil est supposé magnétiquement isotrope.

** Un phénomène réciproque dans un certain sens à celui, qui vient d'être signalé, a été étudié par Harrison et ses collaborateurs (1), Webb (2) et Gorelik et ses collaborateurs(3): c'est la variation sous l'action du champ longitudinal de la composante transversale de l'induction, excitée par le champ du courant parcourant le fil; par suite de cet effet l'impédance du fil dépend du champ longitudinal.

La fonction $f(H)$ (courbe d'aimantation) peut être avantageusement représentée par l'expression approchée

$$f(H) = \frac{\mu_0 H}{1 + k\sqrt{H}}, \quad k = \frac{\mu_0}{B_s} \tag{2}$$

(μ_0 étant la perméabilité initiale et B_s l'induction de saturation).

D'après (1), (2)

$$B_{\parallel} = B\frac{H_{\parallel}}{H} = \frac{\mu_0 H_{\parallel}}{1 + k\sqrt{H_{\parallel}^2 + H_{\perp}^2}} \tag{3}$$

les indices $\parallel$ et $\perp$ désignant les composantes longitudinale et transversale.

Le flux d'induction traversant la section du fil

$$\Phi = \int_0^a B_{\parallel} 2\pi r\, dr = \int_0^a \frac{2\pi\mu_0 H_{\parallel} r}{1 + k\sqrt{H^2 - H_{\perp}^2}}\, dr \tag{4}$$

a étant le rayon du fil et r la distance à l'axe.

Le courant étant supposé sinusoidal et sa pulsation assez petite pour que l'on puisse négliger le skin-effet

$$H_{\perp} = \frac{H_a r}{a}\sin\omega t \tag{5}$$

H_a étant l'amplitude du champ à la surface du fil.

D'après (4), (5) la f. e. m. induite dans la bobine

$$u = -N\frac{d\Phi}{dt} \tag{6}$$

(N étant le nombre de spires) est de pulsation 2ω. Appliquant cette f. e. m. (éventuellement après amplification) à l'entrée d'un instrument de mesure

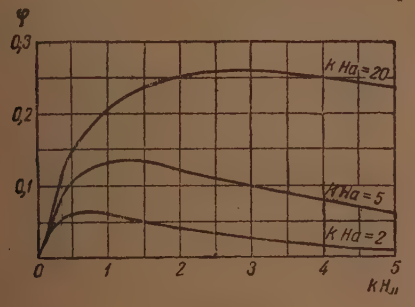

Fig. 1

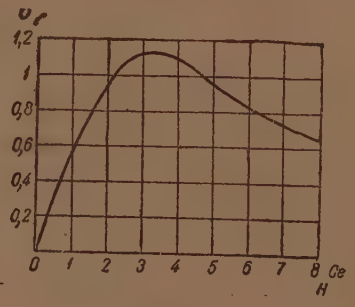

Fig. 2

indiquant la valeur absolue de la moyenne arithmétique, on observera une déviation proportionnelle à

$$U = \frac{2}{\pi}\int_0^{\pi/2} ud(\omega t) = \frac{2\omega N}{\pi}|\Phi|_{\pi/2}^{0} \tag{7}$$

Introduisant (4), (5) dans (7) nous trouvons après calcul

$$U = 4\omega N a^2 B_s \varphi(kH_{\parallel},\ kH_a) \tag{8}$$

où

$$\varphi(kH_{||},\ kH_a) = kH_{||}\left\{\frac{1}{2(1+kH_{||})} + \right.$$

$$\left. + \frac{1}{k^3H_a^2}\ln\frac{1+\sqrt{k^2H_a^2+k^2H_{||}^2}}{1+kH_{||}} - \frac{\sqrt{k^2H_a^2+k^2H_{||}^2}-kH_{||}}{k^3H_a^2}\right\} \qquad (9)$$

La fig. 1 représente φ en fonction de $kH_{||}$ pour quelques valeurs de kH_a. La fig. 2 reproduit une courbe expérimentale de U en fonction de $H_{||}$. On constate un accord satisfaisant entre la théorie et l'expérience, sauf pour les très petites valeurs de $H_{||}$. La f. e. m. résiduelle existant lorsque $H_{||}=0$ peut être attribuée soit à un défaut de l'ajustage du fil dans la bobine, soit à l'existence de tensions anisotropes dans le fil*.

L'oerstedmètre dont il a été question au début utilise la variation de U en fonction de $H_{||}$. Nous avons construit un instrument possédant deux intervalles de mesure: de 0 à 0.6 Oe et de 0 à 2 Oe. Il mesure, naturellement, la projection du champ magnétique extérieur sur l'axe du fil et permet d'en déterminer le sens.

Je tiens à remercier ici le prof. G. Gorelik pour ses conseils.

Institut Physico-Technique.
Université de Gorki.

Manuscrit reçu
le 8. II. 1944.

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* D'après la théorie développée ici l'action sur la bobine du courant alternatif parcourant le fil est, lorsque $H_{||}=0$, purement paramétrique: le courant fait seulement varier périodiquement l'inductivité de la bobine, sans induire de f. e. m.

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CHEMISTRY

CHEMICAL REACTIONS EFFECTED BY GAS IONS IN ELECTROLYTES

By V. I. PAVLOV

(Communicated by V. G. Chlopin, Member of the Academy, 21. I. 1944)

In an earlier communication [1] I have described a method for the production of large currents (0.1 A) through an electrolyte by using the so-called gas electrode under atmospheric pressure. Observations on the chemical action of the gas ions penetrating into electrolyte under these conditions have shown that even with rather weak currents (2—3 mA) the results of the ensuing chemical reactions can be revealed with sufficient clearness both qualitatively (change of colour, precipitation) and quantitatively (titration of solutions, analysis of precipitates).

For the production of stable currents of this order of magnitude at a constant potential difference of 1000 to 2000 V there is no need in a large number of pointed electrodes or a radioactive preparation thas would facilitate the passage of sparks. The discharge going on through the gas at atmospheric pressure from one pointed electrode to the electrolyte can assume the character of a high voltage arc and ensure very stable regime with an arc length of 1—2 mm.

In course of the investigation of this phenomenon a special kind of electrolyser has been devised, consisting of four communicating glass tubes. In the end tube, one side open, one of the electrodes was put in, namely that which was dipped into the electrolyte. In the second tube in the atmosphere of the gas the second electrode was placed over the surface of the electrolyte. The distance between the latter and the electrode (the spark length) was kept constant by changing the level of the liquid in the third tube, this change being effected by raising or lowering the fourth tube (open on one side) which was connected with the third by means of a rubber tube. The gas was admitted in bubbles through the open end of the electrolyser.

Generalizing the results of the varied experiments, one may establish three types of participation of gas ions in the chemical alteratons observed in electrolytes.

I. a) Removal of positive charges from catons, or a reduction reaction (bombardment by —ions); such is the intermediate stage in the production of alkali in a NaCl solution, the formation of CuCl in a $CuCl_2$ solution, the creation of colloidal particles of Cu and Pb, *etc.*

b) Removal of negative charges from anions, *i. e.* an oxidation reaction (bombardment by + ions); such is the formation of MnO_2 with separation of oxygen in a $KMnO_4$ solution, the formation of Turnbull's blue with separation of CN in a $K_4Fe(CN)_6$ solution, *etc.*

This type of interaction between the ions is quite analogous to the recombination of gas ions with the subsequent separation of two particles.

II. The production of complex molecules as a result of the combination of gas ions with electrolyte ions; such are the formation of hydrogen peroxide in water (bombardment by oxygen ions), the oxidation of hydroquinone into quinone, of glucose into gluconic acid, the formation of insoluble precipitate in solutions of $CuSO_4$, $Cu(NO_3)_2$, *etc.*

III. The penetration of gas ions into the electrolyte with conservation of their individuality, whereby the electrolyte ion is replaced by the gas ion of corresponding sign or, in other words, a new compound dissociated into ions is formed; this is exemplified by the production of dissociated manganic acid upon absorption of H^+ (from the gas) by a $KMnO_4$ solution, the combination of Cu^{--} and $2O_2^-$ ions in the case of absorption of O^-_2 (from the gas) by a solution of $CuCl_2$, *etc.*

The last two types of behaviour of gas ions in an electrolytic medium are a novelty and are characteristic of the described method of investigation.

So it may be stated that the gas ions absorbed by an electrolyte in an electrolytic process carried out with a gas electrode are quite peculiar chemical agents meeting in the electrolyte with a varying fate and producing widely different specific chemical reactions, according to the energy involved in the process. This method of carrying out chemical reactions has unquestionable advantages over the ordinary electrolysis with one electrode in the case of Geissler discharge in saturated water vapour. It has also a wider scope and is more profitable in respect to the yield of the reaction, the variety of agents involved and the distinctness of the effect, as compared to the chemical reaction carried out by means of radioactive radiations.

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CHEMISTRY

PRODUCTION OF H_2O_2 BY ELECTROLYSIS OF WATER IN OXYGEN CARRIED OUT WITH ONE ELECTRODE

By V. I. PAVLOV

(Communicated by V. G. Chlopin, Member of the Academy, 8. II. 1944)

A rather long time has elapsed since it was recorded for the first time (1) that when in an electrolysis carried out with one electrode the gas electrode serves as anode and a glow discharge in saturated water vapour takes place, the presence of hydrogen peroxide is observed in the investigated solution. Within recent years the production of H_2O_2 in weak (0.05 n) solutions of H_2SO_4, $HClO_4$, H_3PO_4, NaF, K_2SO_4, KH_2PO_4 and $Na_2S_2O_6$ under the just mentioned conditions has been proved by the experiments of Glasstone and Hickling (2), in which different analytical methods were used. Those authors call attention to the fact that the formation of hydrogen peroxide stops when the direction of the current is reversed and is therefore a typical feature of the gas a n o d e electrolysis, for it does not take place at all when a gas c a t h o d e is used. The equation of the reaction is not mentioned by the authors, yet the necessity of the formation of an H_2O_2 molecule by combining two discharged ions of the hydroxyl is stated quite definitely. At the same time it is possible, in my opinion, to assume that the reaction of the process follows the equation

$$OH^- + H_2O^+ = H_2O_2 + \tfrac{1}{2}H_2$$

In my experiments a pure water, distilled twice, instead of weak water solutions (used for imparting the electric conductivity to the water), was subjected to electrolysis; and ions of oxygen were used as agents to bring about ionic reaction. The method employed in these experiments described earlier (3) secured a stationary regime of the spark which was of constant length, under atmospheric O_2 pressure with a current of several mA and a constant potential difference of about 2000 V. It was possible also to measure the absorption of the gas, which in every experiment amounted to several cubic centimeters, when several tens of coulombs flowed in the circuit. The appearance of H_2O_2 in the liquid tested was qualitatively detected by a reaction with titanic acid, and determined quantitatively by means of titration with 0.01 n solution of $KMnO_4$ in an acid medium.

After a series of experiments it has been found that hydrogen peroxide appears both when the water is bombarded by positive oxygen ions (O_2^+ and O^+), the a n o d e being the gas electrode, and when it is bombarded by the negative ions (O_2^- and O^-), the gas electrode being used as c a t h o d e. In the latter case the yield of H_2O_2 as referred to coulombs was approximately seven times as low as in the former case. In both cases the concentrations of the hydrogen peroxide solutions obtained were of the order of 0.01 n. It may be remarked that

if the water is bombarded by negative ions of hydrogen, an evolution of the gas takes place and there is no formation of H_2O_2.

On the basis of Weiss' ([4]) work I suggested as a possible scheme of reaction the primary formation of HO_2 molecules dissociating with formation of H_2O_2 and separation of oxygen according to the equation $2HO_2 = H_2O_2 + O_2$, or else the formation of hydrogen peroxide according to the equation $H + HO_2 = H_2O_2$. The quantitative verification of the corresponding stcioheiometric equations, connecting the total absorption of gas (difference between absorbed and evolved O_2) with the charge transferred and with the amount of the hydrogen peroxide formed, was made on the basis of measurements. In this way it has been established that within the accuracy of measurement the volume of the absorbed oxygen corresponds well to the course of the reaction and to the amount of transferred electricity. As for the yield of hydrogen peroxide, it proved to be several times below the calculated one. This apparent divergence should undoubtedly be accounted for by the fact that, while creating the hydrogen peroxide through the reaction of oxygen ions with the ions of water, we failed to take into account its annihilation in the process of electrolysis. To make allowance for this loss, the knowledge of the dissociation constant of $H_2O_2 \rightarrow H^+ + HO_2^-$ is required, and no reliable data on this head were at our disposal.

Thus, this particular instance of the application of gas-electrode electrolysis at atmospheric pressure serves to point out the active rôle of new chemical agents. In any electrolyte this rôle may be played by any gas ions introduced, while the process of electrolysis is carried out with one electrode.

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PLANT PHYSIOLOGY

CONTRIBUTION TO THE THEORY AND PRACTICE OF USE OF NITROGEN FERTILIZERS

By M. Ch. ČAJLACHJAN

(Communicated by A. N. Bach, Member of the Academy, 15. XII. 1943)

Both agricultural technique and methods for raising agricultural crops aim at creating such environmental conditions as would make it possible to enhance the growth of plants and to secure at the same time their earlier ripening.

However, both under natural conditions and under conditions of experiment one may not infrequently observe such cases where invigorated vegetative grówth is accompanied by a considerable lagging in the plants' ripening; or else, early ripening is attended by a reduced rate of growth, and the yield turns out to be lower. This led some writers to theoretical generalizations, as a result of which the «theory of antagonism» came into being; this theory affirms that where a trend towards vegetative growth is observed, there cannot possibly be any accelerated development, and contrariwise, if there is a tendency towards reproductive development, a reduction of vegetative growth should be anticipated.

The «theory of antagonism» had subsisted but for a very short while, for it was soon discarded by numerous data secured from work done on vernalization and photoperiodism of plants (1, 2).

In the theory and practice of the use of mineral fertilizers a similar viewpoint exists with regard to nitrogen fertilizers; the latter, especially if there is nitrogen in excess, are believed to enhance the growth and increase the yield of plants, but at the same time to reduce their fruiting and ripening considerably. On account of numerous observations made both under experimental and natural conditions, and while growing agricultural crops, this regularity has been entered into theoretical textbooks and practical manuals on the use of mineral fertilizers (3).

In our work (4) which was carried out after the method of sand-cultures in glass vessels, under conditions of a special light regime, we were able to show that for a whole array of plants food enriched with nitrogen, and consequently, the use of nitrogen fertilizers, too, should be considered as a factor accelerating their fruiting and ripening. At the same time the accumulation of dry mass went on at the same level. Thus, for one group of plants we could observe a most favourable combination of the two chief processes underlying the yield of a plant, *viz.* accelerated ripening and enhanced growth, and accumulation of dry mass. However, the possibility of this combination was not definitely proved, for the yield of seeds was not estimated separately, and only the total dry mass of each plant was weighed. Besides it was necessary to secure this favourable combination without having recourse to any special conditions of light, under usual conditions of natural day.

In order to solve this problem, in 1941, at the Institute of Plant Physiology of the Academy of Sciences of the USSR, experiments were carried out with the following plants: 1) oats, variety «Pobeda», 2) white mustard, 3) blue lupine, 4) millet «Western China», 5) soja «Illini», 6) southern hemp. All these plants were sown within the period 21—22 of May, save soja which was sown on the 29 of May. The plants were sown in metallic vessels 7 kg in volume, into washed sand, and given full Prianishnikov's nutrient solution, except nitrogen. With respect to the latter, all plant species have been divided into two sets, the one of which received $^1/_4$ of the normal ration, and the other 2 normal rations. The plants were grown five per pot, each experiment being run in three replicates; the humidity was maintained at 70 per cent to the total moisture capacity of the sand. The plants in vessels were kept on trucks in a glasshouse under natural day conditions throughout the experiment; the only exception was a two-week period, during which the plants of millet, soja and hemp were being kept under short-day

Fig. 1

conditions, in order somewhat to accelerate the development of those plants, and to make the experimental conditions similar to those under which were grown the southern varieties of the same plants.

After a month the plants grown on different nitrogen fertilizers already displayed marked distinctions: those of them which were given a double nitrogen ration, showed, as a rule, a deep green colour of leaves, while those given a quarter of the normal nitrogen ration were light-green; their culms were thicker and the leaves were in most cases larger in size.

Somewhat later the development of the two sets assumed a sharply different course.

From Table 1 it may clearly be seen that part of the plants, *viz.* oats and mustard, obey the rule adopted in the theory and practice of agriculture, their development being delayed under copious nitrogen food. Others, *viz.* lupine and millet, refute this rule, showing an earlier flowering and a higher general rate of development when given a diet rich in nitrogen; others again (soja and hemp) do not respond to the alteration in their nitrogen ratio, and pass to the phase of flowering and fruiting at the same terms, irrespective of the ration of nitrogen given to them.

In this juncture it should be emphasized that nitrogen food continues to act upon the rate of the plant's ripening in the same direction as it did upon its flowering: in oats and mustard plants full ripening of seeds set in much earlier when the plant is given a quarter of the normal nitrogen ration, than when given a double ration. Unlike this, in millet and lupine, ripening set in sooner in plants that were receiving a double portion of nitrogen, than in those grown on a quarter of the normal ration.

Table 1

Development of Plants as Dependent on Nitrogen Food

	Oats	Mustard	Lupine	Millet	Soja	Hemp
	Panicle formation	Budding	Flowering	Panicle formation	Flowering	Flowering
Nutrient solution + $^1/_4$ N	10.VII	14.VI	24.VII	14.VII	18.VII	14.VII
Nutrient solution + 2 N	14.VII	22.VI	16.VII	11.VII	17.VII	14.VII

In leguminous plants, lupine and soja, there was observed a direct influence of nitrogen food, for in no case did we observe nodules to form upon the roots of these plants.

Alongside of the changes in the development of the plants, nitrogen diet was found to affect also their growth and productivity. Data bearing upon growth and yield are given in Table 2.

Table 2

Growth and Yield of Plants as Dependent on Nitrogen Food

Plant	Height of plant by the end of experiments (cm)		Dry weight of one plant (in g)		Weight of seeds from 5 plants per 1 vessel (in g)		Experiment complete
	$\frac{1}{4}$ N	2 N	$\frac{1}{4}$ N	2 N	$\frac{1}{4}$ N	2 N	
							$\frac{1}{4}$ N 16.VII
Oat «Pobeda» . .	59	76	3.97	5.07	3.57	>3.57	2 N 10.IX
White mustard .	29	39	1.09	4.81	—	—	8.VIII
Blue lupine . . .	31	52	1.78	9.67	0.23	9.97	10.IX
Millet «Western China»	38	62	2,12	7.77	3.46	17.19	16.VIII
Soja «Illini» . .	18	18	1.40	5.34	1.02	9.56	10.IX
Southern hemp . .	37	50	1.15	5.85	—	—	8.VIII

From Table 2 it is seen that as the nitrogen ration is brought from a quarter of the normal up to a double ration, all the plants show a higher rate of growth, a more abundant accumulation of the dry mass and a larger yield of seed.

The effect of nitrogen food upon the growth and development of plants is clearly brought out in Figs. 1 and 2.

In Fig. 1 we see oat plants Nos. 150 and 148 grown on a full nutrient solution added a double ration of nitrogen. The plants are tall, they display a vigorous tillering and bear a large mass of culms and leaves; the fruits on different shoots vary as to ripeness. The oat plants Nos. 132 and 130 received a full nutrient solution, plus a quarter of the normal nitrogen ration. They fail to tiller, are shorter and somewhat withered; the fruits are at the phase of full ripeness (photos taken on August 11, 1941).

Fig. 2 portrayes millet plants Nos. 140 anb 141, grown on a full nutrient solution with addition of a double normal nitrogen ration. The plants are tall, with large numerous panicles upon many of the shoots. Plants Nos. 121 and 123 were grown on a full nutrient solution plus a quarter of the normal nitrogen ration. The plants are short, tillering poor, with scarce panicles (photo taken on July 23, 1941).

A comparison of data on the rate of ripening and the quantity of seed yielded by millet and lupine plants makes us conclude that in the case of these plants, as

well as of some others, a diet enriched with nitrogen may be turned to profit in controlling the two fundamental processes which determine the yield of a given crop.

Fig. 2

On the ground of the experiments described in the present article and in the preceding one (4) millet, perilla, lupine and lettuce should thus be referred to such agricultural crops as can be controlled by means of nitrogen food. There is full ground to believe that the use of nitrates is advisable in the case of buckwheat, soja and hemp, since here the increase of yield is attained without any delay in ripening. Naturally enough, the list of crops here referred is limited in accordance with the work completed heretofore; their number will surely be increased as the investigations will be extended to other cultures.

All what has been said here enables us to introduce the following alterations into the agricultural practice: instead of the actually adopted rule: «in the case of nitrogen excess, the effect ot nitrate fertilizers is to slow down the passing by plants of separate development phases» (3), it should be put: in the case of nitrogen excess, nitrate fertilizers delay the passing of separate phases of development by some plants and, on the contrary, accelerate the development of others.

This conclusion has until now been omitted both by students and practical workers apparently because of the fact that the effect of nitrates was mostly observed on field-crops (wheat, barley, rye), or on other agricultural plants whose response to additional nitrogen food is similar to that shown in our experiments by oats and mustard.

The regularities revealed by us make it necessary to revise the behaviour of a number of agricultural plant species under different conditions of nitrogen food. When working at this problem, it should be borne in mind that high doses of nitrogen fertilizers cannot be applied unless the ontogenetic development of the plants be duly taken into account, and their supply to the plant regulated in accordance with the phase of development.

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EVOLUTIONARY MORPHOLOGY

AGENTS RESPONSIBLE FOR *ACHONDROPLASIA FOETALIS*

By A. N. STUDITSKY

(Communicated by I. I. Schmalhausen, Member of the Academy, 3. IV. 1944)

The disturbance in the development of the embryonic skeleton, known by the name of *Achondroplasia foetalis*, is an affection whose aetiology has still remained enigmatic, and this in spite of the fact that it has been the object of numerous investigations and is now sufficiently well known morphologically (1). The interest for this disease has recently been revived on account of an analogous affection of the skeleton, discovered in chick embryos (2). Landauer studied the structure of the skeleton (3) and of the glands of internal secretion (2,4) in embryos affected with chondrodistrophy. The only conclusion at which he arrived, however, was that the endocrine organs have no direct bearing upon the occurrence of foetal achondroplasia in chicks. In the subsequent, on the ground of experiments carried on in co-operation with Fell on the cultivation of limb anlages *in vitro*, Landauer explained the phenomena of embryonic underdevelopment of the skeleton by a general delay in growth, brought about by some unknown agents. Within recent years Landauer's opinion has found confirmation on the part of Fugo and Witschi (5). In an analysis of their experiments with hypophysectomized chick embryos, these authors have shown that the absence of the pituitary body and the underdevelopment of the thyroid connected herewith are attended by no disturbances in the development of the skeleton.

This problem can now be viewed at a new angle owing to the experiments involving transplantation of the anterior lobe of the pituitary body to the chorioallantois of a chick embryo. As shown by our earlier investigations (7), the effect of a transplantation of the anterior lobe is expressed in the organism of the recipient embryo in a number of alterations; the most salient among them are typical of the thyroid and the parathyroid gland. Here belong also disturbances in the development of the skeleton, and it is with these that the present article is concerned.

The material which served for the work was derived from experimental grafts of embryonic pituitary glands (about 80 operations), from grafts of fragments of anterior lobes of adult chicks (134 operations), and from grafts of hypophysis fragments of various laboratory mammals, chiefly rats (144 operations). The influence produced by the graft upon the embryo lasted 2—10 days, beginning with the moment of operation (made on the 7th—8th day of incubation) till the moment of fixation of the graft and of the organs of the embryo (on the 11th or 13th day of incubation).

The analysis of the development of the embryos under experiment showed a delay in skeletal growth to be the most peculiar effect of the grafts. This delay

reflects the lagging of the general growth of the test-embryos. On the 13th day of incubation in one of the sets the mean weight of nine test-embryos was 3.82 g, while the ten control embryos weighed 4.41 g. In another experimental set, on the 16th day of incubation the mean weight of the thirteen test-embryos was 5.93 g, against 9.57 g of the ten controls.

Still more illustrative is the difference in weight between the individual bones of the skeleton, for example, in the tibia which is subject to most important variations in the course of development.

The outward appearance of the experimental embryos sharply deviates from the normal. Moreover, operating in winter, I succeeded in obtaining also characteristic chondrodistrophic disturbances in the skeletal development, which lead to a typical embryonic achondroplasia (Fig. 1). Aside from their reduced body di-

Fig. 1. Chondrodistrophic micromelia of chick embryos, induced by chorio-allantoic grafts of the anterior lobe of the pituitary body

mensions, chondrodistrophic embryos are distinguished by an extreme delay in their skeletal growth and by a low rate of calcification of the skeleton. As a result of muscular tonicity, this leads to a curvature of their long bones, in the first line of the tibiae. The cartilaginous anlages of the long bones of chondrodistrophic embryos are distinguished by a poor differentiation of cells, by a lack of the typical zones of rounded, disk-shaped and vacuolate cells, and by a degeneration of the intercellular substance of the cartilage. Malformations of the beak, shortening of the skull, eye defects *etc.* are of frequent occurrence.

One may also come across spontaneous chondrodistrophic disorders in the development of the skeleton in chick embryos; thus, a skeleton may display a wide range of a developing chondrodistrophy, beginning with affections of separate sections of the intercellular substance of the cartilagenous bone anlages, and inclusive of typical chondrodistrophic malformations of the skeleton. All these phenomena, however, are observed only during the winter season. The most severe skeleton affections, involving underdevelopment of limbs, micromelia of the embryos, which leads to their death before hatching, occurs no more than in 3—5 cases per 100 embryos, and this with brood-hens kept under most unfavourable conditions, in winter. In our experiments with chorio-allantoic grafts of the anterior lobe of the hypophysis the development of typical chondrodistrophic micromelia was obtained in 20—27 embryos to 100 operations. The experimental origin of the cases of chondrodistrophy obtained is thus beyond any doubt.

What is then the mechanism of the spontaneous chondrodistrophy, and of the one experimentally induced? A direct inhibitory action of the hypophysis grafts upon the skeleton's development under experimental conditions seems scarcely probable, since in all the grafts the eosinophil apparatus which is supposed to excrete the growth hormone is preserved, and it is known to produce an opposite effect. Moreover, making good use of the clear-cut localization of eosinophil ele-

ments in the chick's pituitary body, I succeeded in grafting to the chorio-allantois fragments of the anterior lobe, which contain almost exclusively eosinophil elements, *i. e.* such cells as are connected with the production of the growth hormone. The effect of these grafts did not differ from that of the grafts of other portions of the anterior lobe of the pituitary body. The action produced by the growth hormone of the pituitary body seems to be suppressed in our experiments by the action of other humoral agents. No more probable is the existence of a relation between the chondrodistrophy and the activity of the thyroid. The most intense thyrotropic reaction is observed in chick embryos under the influence of grafting with embryonic pituitary glands; in some cases the reaction attains an extreme intensity ([8]). However, in my experiments with chorio-allantoic grafts of embryonic pituitary lobes I scarcely ever happened to observe the development of micromelia. Neither can the chondrodistrophy be due to a depression of the thyroid.

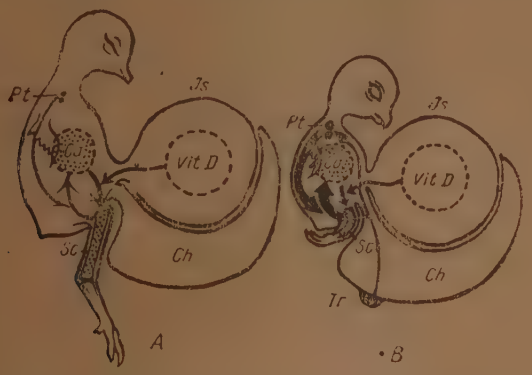

Fig. 2. Scheme of interaction between humoral agents of calcification under normal (*A*) and experimental (*B*) conditions. *Ca*, calcium of internal medium of organism; *Pt*, parathyroid gland; *vit. D*, vitamin D; *Js*, yolk sack; *Sc*, skeleton; *Ch*, chorio-allantois; *Tr*, graft of anterior lobe of pituitary body

which might be induced by high doses of the thyrotropic hormone, for almost in all cases of chondrodistrophy there is a clearly pronounced excitation of the thyroid. Besides, experiments by Fugo and Witschi ([6]) on hypophysectomized chick embryos show that an underdeveloped state of the thyroid does not tell upon the development of the embryo's skeleton.

Thus, there remains the parathyroid gland which under the influence of the chorio-allantoic grafts of the pituitary gland also shows some signs of excitation, *viz.* liquefaction of the cell protoplasm, attended by an accumulation of lipoid inclusions. It may be assumed that its excitation which is undoubtedly related with the excretion of parathohormone induces the mobilization of calcium from the skeleton, in a way analogous to what we know about the effect of high doses of parathohormone upon the growing bone in the post-embryonic development of mammals ([9]).

The fact that the development of a chondrodistrophy induced by the chorio-allantoic grafts of the pituitary body attains its maximum during the winter season, makes one conclude that the effect produced by the excited parathyroid upon the skeleton is controlled by some agents of seasonal nature. The most probable here is the influence of vitamin D whose content in the internal medium of the organism varies strongly along with seasonal fluctuations of the ultra-violet radiation. The rôle of vitamin D, as an agent operative in the skeleton's development, has been thoroughly investigated. Vitamin D promotes the assimilation of calcium, retaining it in the organism and maintaining an optimal ratio between calcium and phosphorus; besides, its effect is to stimulate the supply of calcium from the internal medium of the organism on to the skeleton.

The effect of parathohormone, which tends mainly to keep the quantity of calcium in the internal medium of the organism at an optimal level, stimulates the flow of calcium in an opposite direction. Thus, vitamin D and the hormone of the parathyroid which are in many respects supplementing each other, may act as antagonists as regards the ossification and calcification of the developing skeleton. Under optimal developmental conditions, the yolk of a hen's egg is so rich in vitamin D that the excitation of the parathyroid does not interfere with the depositing function of a calcifying skeleton. In winter the amount of vitamin D in the yolk of a hen's egg is reduced to one fifth of the summer amount ([10]). In some cases this amount may prove insufficient to control the calcium equilibrium between the skeleton and the internal medium of the embryo. In this case a causal excitation of the parathyroid may lead to a fatal calcium deficiency in the skeleton, to a disturbed ossification and to a deformation of bones, resulting in a spontaneous chondrodistrophy. In experiments on the grafting of the pituitary gland to the chorio-allantois there takes place an analogous interaction between the vitamin and the parathohormone excreted by the parathyroid gland, as the basic humoral agents of calcification. This interaction is schematically represented in Fig. 2.

From what has been said above it may be concluded that the achondroplasia of the chick embryo is a hyperparathyroid distrophy of the skeleton, developing against the background of avitaminosis D. And it does not seem improbable that our hypothesis as to the nature of foetal achondroplasia can also be extended to the analogous affection in man.

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PHYSIOLOGY

REACTIVE PROPERTIES OF NERVE CELLS OF FROG HEART

By N. A. SMITTEN

(Communicated by L. A. Orbeli, Member of the Academy, 24. XII. 1943)

Within recent years a whole series of studies concerned in vital microscopic examination of nerve tissue were carried out at the Laboratory of Prof. B. I. Lavrentiev.

These studies involved observations on the isolated frog heart (after the method of Gramenizky), as well as microscopic examinations of the nerve elements in mammals, carried out with the aid of a vertical illuminator (ultropak Leitz). The reactive properties of neurons, of nerve fibres and nerve endings were investigated by way of stimulating the corresponding nerve mediators with induced current, as well as by applying a high-freguency field of ultra-short waves. Besides, use was made of mechanical stimulants (microneedle) and als› of certain chemical agents ([3],[6],[9]).

While investigating neurons of the Auerbach plexus in a live rabbit, Lavrentiev and Wishnevsky succeeded in observing a very curious phenomenon. When acted upon with weak acid or alkaline solutions, cooled or heated, as well as when subjected to mechanical trauma, the nerve cells of the intestinal ganglia suffered peculiar alterations. After the stimulants listed above had been applied to the surface of the intestine, a drop of a weak solution of methylene-blue made the neuron take on a colour strikingly similar to typical pictures of the Nissl granulosity. The nucleus appeared clearly outlineable, but remained colourless. In the protoplasm there appeared granules stained blue and green. Their arrangement corresponded to a typical pattern.

I undertook an attempt to check up this phenomenon, and to secure more precise data as to the conditions under which it arises, using as material live neurons of a frog heart.

Live neurons were examined under the microscope in the heart of a frog, distended according to Gramenizky's procedure. The heart of a frog, isolated together with the head and having thus preserved its nerve connexions, was placed upon the stage of a microscope. The original idea was to investigate the response of live neurons to the action of muriatic acid without a subsequent staining. With this aim in view, a drop of a weak HCl solution (from 0.05 *n* to 0.1 *n*) was applied to the preparation. Immediately after this, the ganglia of the heart were examined under various magnification. One of the first responses of the nerve cell to the stimulus was the appearance of a distinct structure of the nucleus. As a rule,

it cannot be detected in a living, uninjured nerve cell. Thus, we obtained here a typical Nassonov paranecrosis which had earlier been reported by Makarov (7). The cytoplasm of the neuron was found to display at the same time a well expressed granulosity. Some of the cells, however, showed no granular texture, but instead their protoplasm assumed a peculiar lustre, against the background of which the nucleus looked sharply outlined (Fig. 1). Subsequent examination showed that in one and the same preparation various neurons are apparently present in different functional states, responding in a different way to the action of the acid.

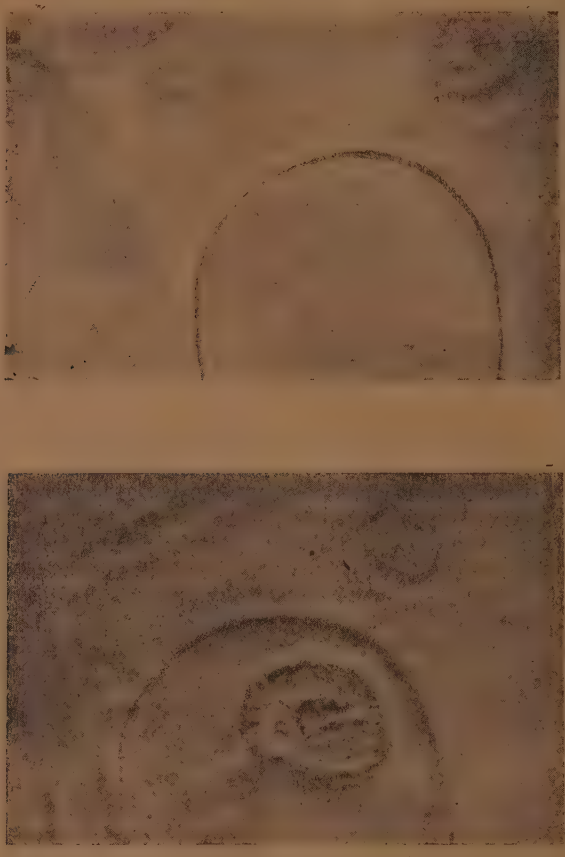

Fig. 1. Appearance of nucleus and nucleolus in a nerve cell acted upon with a 0.05 *n* HCl solution: *a*) prior to treatment, *b*) after treatment

In some cases abrupt shifts in the structure of the neurons set in upon the application of two drops of a 0.05 *n* HCl solution. In other cases this concentration failed to produce any visible alteration in the neurons; in order to bring about changes in the nucleus and in the protoplasm, the concentration of the HCl solution had to be raised to 0.1 *n* and higher.

It was the staining with methylene-blue (1—2 drops of a 0.5—1 per cent solution) that was found helpful in finding out the difference between the states of the neurons.

The partition between the auricles of the heart was thereby found to be made up of three kinds of cells: of cells with both nuclei and cytoplasm stained diffusely, which is known to characterize killed cells; of structureless and colour-

less cells which seem to be more stable; and, finally, of cells whose cytoplasm consists of granules stained blue and green, while their nucleus is colourless (Fig. 2). The latter picture is in full agreement with that observed by Lavrentiev and Wishnevsky in live neurons of the rabbit intestine. There was a striking resemblance between the size and arrangement of the granules in their preparations, and those in preparations stained after Nissl.

In order to become sure of the identity of the two pictures a preparation of a live frog heart with neurons stained with methylene-blue was fixed with a saturated solution of ammonium molybdate. Upon washing, the preparation was cut on a freezing microtome. The sections were dehydrated and embedded in Canada balsam.

After having been thoroughly examined under high magnification, the neurons subjected to vital staining with methylene-blue proved to have exactly the same appearance as if upon fixation they were stained according to Nissl.

We are thus in position to assert that the phenomenon of tigroid staining according to Nissl can be reproduced *in vivo*. The velocity at which this phenome-

Fig. 2. Nissl's type of staining after treatment with HCl

non arises in living nerve cells makes us doubt whether we really have to do here with a separation of granules. If this be true, we should certainly observe a series of forms transitional between neurons with a small number of granules and the ultimate picture offered by a neuron stained according to Nissl. As a matter of fact, this has never been observed, and the neuron displays at once the typical Nissl pattern, or else defies staining at all.

For the interpretation of the mechanism of the phenomenon here described, further investigation is necessary. Certain assumptions may, however, by advanced at the time being. As follows from observations by Lavrentiev, Wishnevsky and the present author, the neurons assume Nissl's type of pattern in response to a variety of agents (shifts in the pH, mechanical trauma, heating, cooling). This non-specific response is typical of irreversible changes of the protoplasm, for which Nassonov has suggested the name of paranecrosis.

It seems highly probable that the colour reaction described by us presents one of the forms of the neuron's paranecrotic state. If this be so, the classical picture of Nissl's staining may be interpreted in the following way. Each time a nerve tissue is put into some fixative (alcohol, formalin, chromium mixtures, *etc.*), the contact of a neuron still alive with the fixative results in paranecrosis which in a nerve cell is accompanied with the outlining of a nucleus and appearance of a granular texture. In the subsequent the fixing liquid makes it preserve this condition.

Conclusions

1) If nerve cells of a living and operating heart of a frog are acted upon with weak solutions of muriatic acid, the cytoplasm of its neurons shows a distinct granular texture and a well-outlined nucleus, the latter being indiscernible in intact cells.

2) The structural alterations of the neurons, caused by muriatic acid, become still more salient after the preparation is stained with methylene-blue. The picture obtained is found to be identical with that offered by the staining after Nissl.

3) The Nissl pattern can thus be obtained by vital staining of neurons.

4) Assumption is made that the Nissl type of staining is an indication to the paranecrotic condition of the nerve cells.

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PHYSIOLOGY

RATE OF RESPIRATION IN MAMMALS AS DEPENDENT ON TEMPERATURE

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1. Introduction. Variation in the rate of breathing is one of the important components of the processes of metabolism control and thermoregulation. As a rule, changes in the environmental temperature are accompanied by changes in the rate of respiration. The simplest relation between the environmental temperature and the rate of breathing is observed in poikilothermal forms in which a rise in temperature causes an increase in the frequency of respiration, while a drop results in a reverse phenomenon ([2]). A similar picture is observed in animals liable to dormancy ([5]).

For homoiothermal animals the relation between the rate of breathing and the temperature of the surrounding medium has been studied in less detail, the study of thermoregulation being largely focussed on records of the variation of gaseous exchange ([3], [4], [11]).

We thought that a study of the dependence of breathing frequency in different forms of mammals upon the environmental temperature was likely to reveal peculiar features of their adaptation to this factor. Therefore a series of observations were carried out at the Moscow Zoological Gardens, the results of which are exposed in the present note. Part of the data here reported have been collected by R. I. Athonskaya, of this Laboratory; a certain amount of material has been obtained by A. L. Ponomarev from the Zoological Gardens and Menagerie in Alma-Ata.

2. Observational Results. Counts of respiration under varying temperature conditions were taken for 19 species of mammals; it stands to reason that in the case of some of them, for example, apes and tropical *Ungulata*, neither excessive cooling, nor overheating could possibly be applied.

In the first line the presence of pronounced distinctive feature should be pointed out in the mode of dependence of the rate of the respiration on air temperature in a number of species within a single taxonomical group.

Thus, for instance, in red and silver-black foxes the effect of a drop of air temperature is to reduce the rate of respiration, whereas in the polar fox cooling causes a rise in respiration frequency, followed by a slowing-down (Table 1 and Figs. 2 and 3). Analogous differences have been observed also for lions and leopards, baboons, shimpansee and macaco (Table 1 and Figs. 2 and 3). On the other hand, there are cases where the mode of response being much similar for species within the same taxonomic group, there are, nevertheless, material quantitative distinctions. Thus, for example, the camel responds to a rise in temperature from $-10°$ up to $+25°$ by increasing its respiration rate 1.5 times only (from 5.4 to 8.6

Table 1

Rate of Respiration in Marsupial and Carnivorous Mammals as Affected by Temperature

Species	Number of individuals	Number of records	Number of respiratory movements within 1 min. according to temperature in °C							
			−20° — −10.1°	−10° — −0.1°	0° +5°	+5.1° + +10°	+10.1° + +15°	+15.1° + +20°	+20.1° + +25	+25.1° + +30°
Macropus rufus	1	57	—	—	18.0	14.0	14.3		11.5	15.0
Alopex lagopus	2	50	15.8	21	31.7		24.4	25.8	22.9	—
Vulpes vulpes	4	71	16.7	19.2	25.8		26.0		28.7	39.0
Vulpes fulva	7	101	—	25.6	23.0		21.2	25.5	29.3	35.5
Thalassarctos maritimus	3	75	5.3	7.3	13.7		14.8		24.6	30.0
Selenarctos hymalayanus	2	96	5.1	5.3	7.7		—	15.4	23.7	40.5
Ursus malayanus	2	76	—	—	—	17.5	29.0	29.0	42.3	75.5
Panthera pardus	4	57	—	—	—	—	13.7	24.4	14.1	36.5
Panthera leo	5	77	—	—	—	12.8	16.0		23.7	42.3

respirations per minute), whereas in the llama a temperature variation within this range makes the temperature frequency become its double.

The three bear species under observation also differed as to the rate of the respiration (Fig. 1 and Table 1).

Table 2

Rate of Respiration in Some *Ungulata* and Apes as Affected by Temperature

Order	Species	Number of individuals	Number of records	Number of respirations within 1 min. according to temperature in °C							
				−20° — −10.1°	−10° — −0.1°	0° +5°	+5.1° + +10°	+10.1° + +15°	+15.1° + +20°	+20.1° + +25°	+25.1° + +30°
Artiodactyla	*Giraffa cameleopardalis*	2	55	—	—	—	7.5	8.5	8.6	10.6	10.3
	Bos indicus	2	44	—	—	—	13	14.3	17.2	—	18
	Poephagus grunniens	1	54	8.9	10.1	10.0		18.5		20.2	37
	Maral—Cervus elaphus sibiricus	4	52	8.9	9.8	12.0	17.5	17.8	39.2	—	—
Tylopoda	*Llama glama*	2	37	11.2	11.5	10.5	—	20.5		20.0	—
	Camelus bactrianus	1	37	8	5.4	6.6		—	8.6	8.6	12.0
Perissodactyla	*Hippotigris zebra*	2	44	—	—	—	13	14.2	20.5	28.4	—
Primates	*Papio hamadryas* and *P. anubis*	6	50	—	—	—	18.6	22.4	24.5	24.3	18.2
	Macaccus mulattus	7	47	—	—	—	—	25.2	22.1	30.6	—
	Pan troglodytes	1	26	—	—	—	15.2	15.0	21.7	18.2	20.0

From the regular distribution of the total material it can safely be concluded that the average figures derived by us are by no means due to a causal coincidence of individual records (cf. data for the polar and the Malayan bear in Fig. 1).

3. Discussion. The data obtained are of unquestionable interest. The first to be emphasized is the regular variation in the mode of the respiration

response to temperature fluctuations, as shown by different species of mammals. It was in kangoroo alone that we succeeded to reveal a relation between the rate of respiration and the temperature, analogous to the variation in the intensity of gas exchange observed in homoiothermal forms: accelerated respiratory movements both in the case of overheating and cooling, and the presence of a zone of minimum respiration frequency under conditions of «neutral temperature». A similar relation between the rate of respiration, intensity of gas exchange and temperature is also observed in rodents ([3], [6], [11], [13]).

Fig. 2 goes to illustrate the parallelism which has been observed in rodents and marsupials. In the rest of mammals cooling was found to bring about either a reduced respiration rate (Figs. 1 and 3), or a somewhat more complicated reaction, *viz.* after an increase in respiration frequency there followed, as the temperature continued to fall, a slowing down of the rate of respiration (Fig. 2).

Finally, the type of reaction that has been recorded for polar foxes, leopards, baboons and shimpansees, occupies a kind of intermediate position between the two extremes. In the forms characterized by a reduced rate in response to cooling a series of transitional types can also be observed (cf. data for foxes and bears).

Now let us come back to the quantitative distinctions between the reaction studied in species belonging to the same taxonomic group. It can well be said that in a number of other species, along with a direct relation between the respiration rate and the temperature of the medium, the reaction varies sharply as to its expression. Thus, for example, in the Malayan and Hymalayan bears, in the lion, zebu, maral (Siberian red deer), the respiration frequency increases at a higher rate than in the polar bear, llama, zebu, yack and macacoes. Some species (giraffe, camel) are characterized by an almost constant rate of respiration within an extremely wide range of temperature.

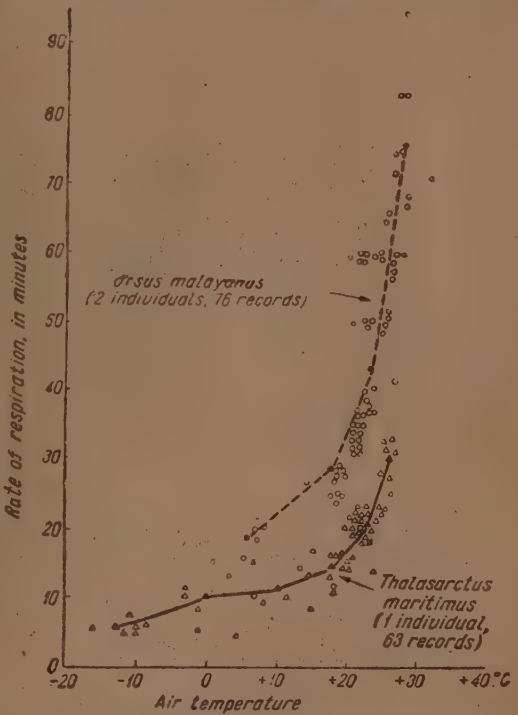

Fig. 1. Rate of the respiration in Malayan and polar bears as dependent on temperature (means and results of individual observations)

All these data give us ground to assume that the mechanism controlling the intensity of gas exchange greatly differs in various species of mammals.

It appears that while in rodents* and kangaroo the increased intensity of gas exchange in response to cooling seems to be connected with a higher frequency of breathing, in other mammals enhanced oxygen consumption proceeds at the expense of deeper respiratory movements ([4], [10], [12], [13]).

The transitional type of the reaction, *viz.* increased rate of respiration in response to slight cooling, and slowing down as the temperature continues falling, seems to be due to a consecutive co-operation of the two mechanisms of gas exchange control, increased rate of breathing and increased depth of inspirations.

All these data go once more to stress the import of the thesis advanced by Barcroft ([1]), *viz.* that «every adaptation is an integration».

* Herter (1940) observed an increase in the rate of respiration in the case of cooling also in dormouse, bank vole and grey and white mice.

If our data should be interpreted to mean that the most important physiological reaction of homoiothermal animals, *viz.* the increase in the gas exchange intensity in response to cooling, is brought about within the class of mammals by way of different relations between the separate components of the respiration process, it is but natural to put the question, What is then responsible for this reaction being different as to the case? In spite of its incompleteness, the material discussed above enables us to conclude that the most primitive type of the reaction, *i. e.* the increased rate of respiration in response both to overheating and cooling, as observed in rodents and kangaroo,

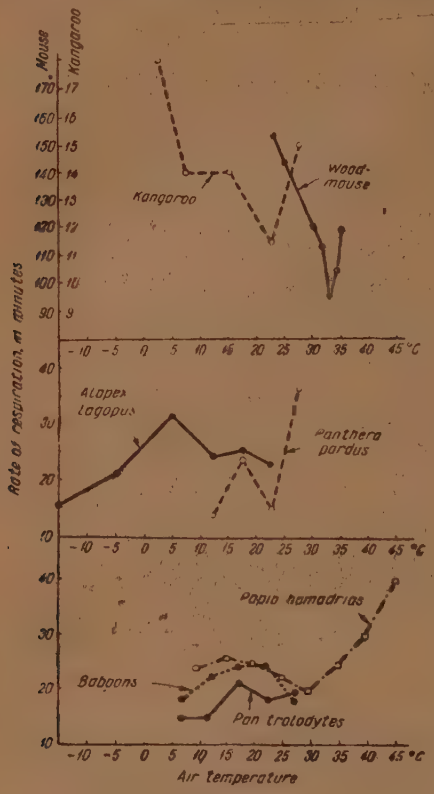

Fig. 2. Relation between temperature and the rate of the respiration in some mammals (data on wood-mouse adopted from Herter, 1940, on baboon from Slonim and Scherbakova, 1939)

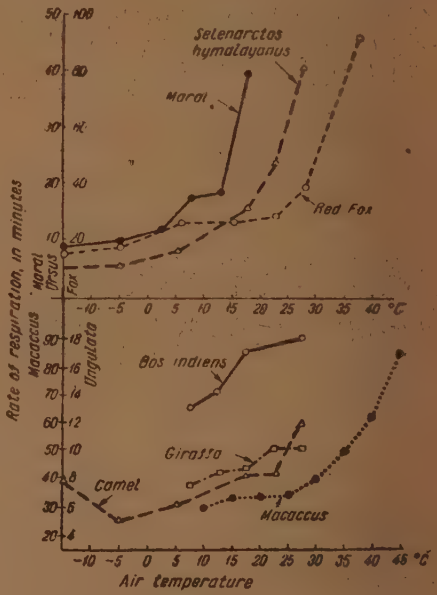

Fig. 3. Relation between the rate of the respiration and temperature in some species of carnivorous and ungulate mammals and apes (data concerning macacoes after Slonim and Scherbakova, 1939)

seems to characterize homoiothermal animals with a well-pronounced chemical thermoregulation. Let us recall that in the guinea-pig the gas exchange increases within a temperature interval between 35—37° and 0—+1.5° by 237 per cent ([11]), in the rat, by 257 ([3]) and in the wood-mouse by as much as 453 per cent ([8]).

Another extreme type of relation between respiration frequency and temperature, *viz.* its being slowed down in response to temperature decrease, seems to be typical of species liable to less important variations in their gas exchange intensity in response to temperature variations of the medium. Thus, in the case of a temperature drop from 25 to 5° the red fox increases its oxygen consumption but by 110 per cent, the bear does so in response to a temperature fluctuation within a range between 25—10°, and the macaco responds to a temperature drop from 40 to 5° (!) by an increase of 120 per cent only ([13]).

Naturally enough, the intermediate type of reaction corresponds to indices of gas exchange of an intermediate type. Thus, in the polar fox the effect of a temperature drop from 15 to 5° is to increase oxygen consumption by 210 per cent, in the baboons and sphinxes a drop from 30—35° to 5° brings about an increase by 140—150 per cent ([13]).

All that has been said above permits to assert that the character studied by us, being an important component of the processes of regulation of metabolism, heat production and heat loss, plays, like many other physiological functions, an absolutely different part not only in animal groups of different taxonomic position, but sometimes in closely related forms within the same group as well. We have already dwelt more than once upon the problem of divergency of physiological characters in closely related forms ([6-9]). The data discussed above go well to confirm the correctness of this opinion.

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